

## THE OBSERVING RESPONSE IN DISCRIMINATION LEARNING<sup>1</sup>

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Recently, the importance of an orienting or observing response has been emphasized in the formulation of a general theory of discrimination behavior (Atkinson, 1958, 1960; Burke & Estes, 1957; Restle, 1959; Wyckoff, 1952). Unfortunately, for many experimental problems it is not clear how such a theory should be formalized. In particular, there are not enough experimental data available to permit a detailed specification of the postulates relating observing responses and such variables as stimulus dimensions, reinforcement schedules, and stimulus schedules. The purpose of this study is to gain information about this class of relations by modifying the typical discrimination task so that observing responses can be categorized and directly measured.

The experimental situation is considered as a sequence of discrete trials. Each trial is described in terms of the following classifications:

*T<sub>1</sub>, T<sub>2</sub>: Trial type.* Each trial is either a *T<sub>1</sub>* or a *T<sub>2</sub>*. Trial type is selected by *E* and determines in part the stimulus event occurring on that trial.

*O<sub>1</sub>, O<sub>2</sub>: Observing responses.* At the start of each trial, *S* makes either an *O<sub>1</sub>* or *O<sub>2</sub>*. The particular observing response made determines in part the stimulus event for that trial.

*s<sub>1</sub>, s<sub>2</sub>: Stimulus events.* Following the observing response, one and only one of these stimulus events (discriminative cues) occurs. On a *T<sub>1</sub>* trial, *s<sub>1</sub>* or *s<sub>2</sub>* can occur; on a *T<sub>2</sub>* trial, *s<sub>2</sub>* or *s<sub>2</sub>* can occur.

*A<sub>1</sub>, A<sub>2</sub>: Discrimination responses.* On each trial *S* makes either an *A<sub>1</sub>* or *A<sub>2</sub>* response to the presentation of the stimulus event.

*E<sub>1</sub>, E<sub>2</sub>: Reinforcing events.* The trial is terminated with the occurrence of one of these events. An *E<sub>1</sub>* indicates that *A<sub>1</sub>* was the correct response for that trial and *E<sub>2</sub>* indicates that *A<sub>2</sub>* was correct.

The sequence of events on a trial is as follows: (a) ready signal occurs and *S* makes either an *O<sub>1</sub>* or *O<sub>2</sub>*; (b) following the observing response *s<sub>1</sub>*, *s<sub>2</sub>*, or *s<sub>2</sub>* is presented; (c) to the onset of *s<sub>i</sub>*, *S* makes either *A<sub>1</sub>* or *A<sub>2</sub>*; (d) the trial is terminated with reinforcing event *E<sub>1</sub>* or *E<sub>2</sub>*.

The trial type and reinforcing event are determined by *E*. The probability of an *E<sub>1</sub>* event on a *T<sub>1</sub>* trial is denoted by  $\pi_1$ , and the probability of an *E<sub>1</sub>* event on a *T<sub>2</sub>* trial is denoted by  $\pi_2$ . Consequently, the probability of an *E<sub>2</sub>* is  $1 - \pi_1$  on a *T<sub>1</sub>* trial and  $1 - \pi_2$  on a *T<sub>2</sub>* trial. The two types of trials are equiprobable in the present experiment.

The particular *s<sub>i</sub>* event that is presented on any trial depends on the trial type and the observing response. If an *O<sub>1</sub>* is made, then (a) with probability  $\alpha$  the *s<sub>1</sub>* event occurs on a *T<sub>1</sub>* trial and the *s<sub>2</sub>* event occurs on a *T<sub>2</sub>* trial, and (b) with probability  $1 - \alpha$  the *s<sub>2</sub>* event occurs, regardless of the trial type. If an *O<sub>2</sub>* is made, then (a) with probability  $\alpha$  the *s<sub>2</sub>* event occurs, regardless of the trial type, and (b) with probability  $1 - \alpha$  the *s<sub>1</sub>* event occurs on a *T<sub>1</sub>* trial and the *s<sub>2</sub>* event occurs on a *T<sub>2</sub>* trial.

To clarify the experimental procedure, consider a case where  $\alpha = 1$ ,  $\pi_1 = 1$ , and  $\pi_2 = 0$ . If *S* is to be correct on every trial, he must make an *A<sub>1</sub>* on a *T<sub>1</sub>* trial and an *A<sub>2</sub>* on a *T<sub>2</sub>* trial. However, *S* can gain information about the trial type only by making the appropriate observing response. That is, *O<sub>1</sub>* must be made in order to identify the trial type; the occurrence of *O<sub>2</sub>* always leads to the presentation of *s<sub>2</sub>*. Hence, for perfect responding in this case, *S* must make the *O<sub>1</sub>* response with probability 1 and then make *A<sub>1</sub>* to *s<sub>1</sub>* or *A<sub>2</sub>* to *s<sub>2</sub>*.

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The aim of this study is to investigate the effect of various event schedules on observing behavior. In particular, we are interested in the values of  $\pi_1$ ,  $\pi_2$ , and  $\alpha$  as determiners of the probability of an  $O_1$  response.

### THEORY

The analysis of the data will be organized within the framework of a Markov chain model which is closely related to stimulus sampling theory as first formulated by Estes (1950) and Estes and Burke (1953). The mathematical techniques for the model considered in this paper have been presented in detail elsewhere (Atkinson, 1960; Suppes & Atkinson, 1960) and the reader is referred there for a rigorous development.

The basic assumption for observing responses is that if  $O_i$  ( $i = 1, 2$ ) occurs and leads to the selection of a stimulus which in turn elicits a correct discrimination response, then  $S$  will tend to repeat that observing response on the next trial. However, if  $O_i$  occurs and leads to the selection of a stimulus which elicits an incorrect discrimination response, then  $S$  will tend not to repeat that observing response on the next trial. Conceptually, this assumption is similar to that proposed by Wyckoff (1952) and Atkinson (1958).

It is next assumed that  $S$  can be described by an ordered four-tuple at the start of trial  $n$  where (a) the first member is 1 or 2 and indicates whether  $O_1$  or  $O_2$  will be made on trial  $n$ , (b) the second member is 1 or 2 and indicates whether  $s_1$  is conditioned to  $A_1$  or to  $A_2$  (i.e., whether  $A_1$  or  $A_2$  will occur if  $s_1$  is presented), (c) the third member is 1 or 2 and indicates whether  $s_2$  is conditioned to  $A_1$  or to  $A_2$ , and (d) the fourth member is 1 or 2 and indicates whether  $s_2$  is conditioned to  $A_1$  or to  $A_2$ .

These four-tuples will be referred to as subject states and assigned identifying numbers as follows:

- |           |           |            |            |
|-----------|-----------|------------|------------|
| 1. (1111) | 5. (1211) | 9. (2111)  | 13. (2211) |
| 2. (1112) | 6. (1212) | 10. (2112) | 14. (2212) |
| 3. (1121) | 7. (1221) | 11. (2121) | 15. (2221) |
| 4. (1122) | 8. (1222) | 12. (2122) | 16. (2222) |

From trial to trial  $S$  may change states depending on the sequence of responses and reinforcements. The possible changes are specified by the following axioms:

Axiom 1: With probability  $\theta'$  the  $s_k$  ( $k = 1, 2, c$ ) stimulus presented on trial  $n$  will become conditioned to the reinforced response; if it is already conditioned to that response it remains so. (For example, if  $s_k$  is presented and followed by  $E_j$  then with probability  $\theta'$  it will become conditioned to  $A_j$ .)

Axiom 2: If  $O_i$  ( $i = 1, 2$ ) is made on trial  $n$  and followed by an  $s_k$  which elicits a correct discrimination response, then  $S$  will repeat the same observing response on the next trial. However, if  $O_i$  is made and followed by an incorrect discrimination response, then with probability  $\theta''$   $S$  will make the other observing response on the next trial.

From these assumptions and the event schedules employed in this experiment, it can be shown that the sequence of random variables which take the subject states as values is an irreducible, aperiodic Markov chain. This means among other things that a transition matrix  $[p_{ij}]$  may be derived from these assumptions where  $p_{ij}$  is the conditional probability of being in state  $j$  on trial  $n + 1$  given state  $i$  on trial  $n$ . The learning process is completely characterized by these transition probabilities and the initial probability distribution on states.

To clarify the application of the axioms we derive one element of  $[p_{ij}]$ . Assume  $S$  is in State 1211 at the start of Trial  $n$  and  $T_1$   $E_1$  is selected by  $E$  with probability  $\frac{1}{2}\pi_1$ . Then an  $O_1$  occurs with probability 1 and an  $s_1$  is presented with probability  $\alpha$ ; to the presentation of  $s_1$  an  $A_2$  is made. The  $S$ 's discrimination response was incorrect and therefore with probability  $\theta''$  the observing response changes from  $O_1$  to  $O_2$ . Also, with independent probability  $\theta'$  the conditioning of  $s_1$  changes from  $A_2$  to  $A_1$ . Multiplication of the conditional probabilities yields the probability of going from State 1211 to State 2111; i.e.,  $p_{13,9} = \frac{1}{2}\pi_1\alpha\theta'\theta''$ .

In this paper, we shall be primarily interested in the asymptotic behavior of

$S$ . Consequently,  $p_{ij}^{(n)}$  is defined as the probability of being in state  $j$  on trial  $n + 1$ , given that on Trial 1  $S$  was in State  $i$ . Then the following limit exists and is independent of  $i$ ,

$$u_j = \lim_{n \rightarrow \infty} p_{ij}^{(n)}.$$

The quantity  $u_j$  can be interpreted as the asymptotic probability of being in state  $j$  no matter what the initial distribution. Experimentally, we will be interested in evaluating the following theoretical predictions:

$$P_{\infty}(O_1) = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 \quad [1]$$

$$P_{\infty}(A_1|T_1) = u_1 + u_2 + u_9 + u_{10} + \alpha[u_3 + u_4 + u_{13} + u_{14}] + (1-\alpha)[u_5 + u_6 + u_{11} + u_{12}] \quad [2]$$

$$P_{\infty}(A_1|T_2) = u_1 + u_5 + u_9 + u_{13} + \alpha[u_3 + u_7 + u_{10} + u_{14}] + (1-\alpha)[u_2 + u_6 + u_{11} + u_{15}] \quad [3]$$

$$P_{\infty}(O_1 \cap A_1) = u_1 + \alpha u_3 + (1-\alpha)u_6 + (\alpha/2)[u_4 + u_7] + (1-\alpha/2)[u_2 + u_5] \quad [4]$$

$$P_{\infty}(O_2 \cap A_1) = u_9 + \alpha u_{14} + (1-\alpha)u_{11} + [(1-\alpha)/2][u_{12} + u_{15}] + [1 - (1-\alpha)/2][u_{10} + u_{13}] \quad [5]$$

Equation 1 gives the asymptotic probability of an  $O_1$  response. Equations 2 and 3 present the asymptotic probability of an  $A_1$  response on  $T_1$  and  $T_2$  trials, respectively. Finally, Equations 4 and 5 present the asymptotic probability of the joint occurrence of each observing response with an  $A_1$  response.

## METHOD

*Experimental parameter values.*—Six groups of  $S$ s were tested. For all groups  $\pi_1 = .9$ . The groups differed with respect to the experimental parameters  $\pi_2$  and  $\alpha$ ; three values of  $\pi_2$  (.9, .5, and .1) and two values of  $\alpha$  (1.00 and .75) were used. Specifically,  $\pi_2 = .90, \alpha = 1.0$  (Group I);  $\pi_2 = .50, \alpha = 1.0$  (Group II);  $\pi_2 = .10, \alpha = 1.0$  (Group III);  $\pi_2 = .90, \alpha = .75$  (Group IV);  $\pi_2 = .50, \alpha = .75$  (Group V); and  $\pi_2 = .10, \alpha = .75$  (Group VI). These particular values of  $\pi$  were

selected because they had been used in a similar discrimination experiment where the observing response was not available (Atkinson, Bogartz, & Turner, 1959).

*Subjects.*—The  $S$ s were 240 undergraduates obtained from introductory courses in psychology. They were randomly assigned to groups with the restriction of 40  $S$ s in each group.

*Apparatus.*—The  $S$ s were run in subgroups of two with each  $S$  seated in a private booth. The apparatus, viewed from within  $S$ 's booth, consisted of a shelf at table level which was 30 in. wide and 13 in. deep. A panel 30 in. wide and 30 in. high was mounted vertically on the edge of the shelf farthest from  $S$ . Four red panel lights (the  $s_i$  stimuli) were in a column and centered on the vertical panel; the bottom light was 20 in. from the base of the panel; the others were spaced above each other at 1½-in. intervals. Two silent operating keys (the  $A_1$  and  $A_2$  responses) were each mounted 1½ in. in from the edge of the shelf facing  $S$ ; these keys were 14 in. apart and centered on the column of red lights. On the shelf, 1 in. behind each of these keys, was a white panel light ( $E_1$  and  $E_2$  events). Two additional silent operating keys (the  $O_1$  and  $O_2$  responses) were each mounted 6 in. in from the rear edge of the shelf; these keys were 2 in. apart and also centered on the red lights. A green light (the signal) was centered 3 in. behind the observing response keys on the shelf. The presentation and duration of the lights were automatically controlled.

*Procedure.*—Within each of the six experimental groups, four subgroups of 10  $S$ s were formed by counterbalancing right and left positions of the observing response and the discrimination response keys. For each  $S$  one of the four red lights was randomly designated  $s_1$ , another  $s_6$ , and another  $s_2$ ; the fourth light was not used.

The  $S$ s were read the following instructions:

The present study is designed to determine how well you can do on a very difficult pattern recognition problem. We run subjects in pairs to save time, but you are both working on completely different problems. The experiment for each of you consists of a series of trials. The green light on your panel will go on to indicate the start of each trial. Some time later, one or the other of the two lower white lights will go on. Your job is to predict on each trial which one of the two white lights will go on and to indicate your prediction by pressing one of the two lower keys.

However, before you make your prediction you will receive additional information. That is, as soon as the green light goes on, press one or the other of the two upper keys—which key you press is up to you. Shortly thereafter, one of the four red lights will go on. The particular red light which goes on depends in part on the key you press. Further, the red light which goes on will help you in making your prediction as to which white light goes on. After you have seen one of the red lights go on, you will then predict which white light will go on by pressing the proper key. That is, if you expect the left white light to go on, press the left lower key, and if you expect the right white light to go on, press the right lower key. If the light above the key you pressed goes on, your prediction was correct, but if the light above the key opposite from the one you pressed goes on, you were incorrect and should have pressed the other key. Thus, for a single trial, the sequence of events is as follows: (1) the green light goes on to signal the start of the trial, (2) you press one of the

two upper keys, (3) one of the red lights will go on, (4) you press one of the two lower keys, (5) if the white light goes on above the key you pressed, your prediction was correct; if the light above the key opposite from the one you pressed goes on, you were incorrect and should have pressed the other key.

Questions were answered by paraphrasing the appropriate part of the instructions. Following the instructions, 200 trials were run in continuous sequence. This sequence was followed by a 5-min. rest period; during this period no questions referring to the experiment were answered by *E*, and *S*s were not allowed to discuss the experiment. Following the rest, 200 additional trials were run. For each *S*, random sequences of  $s_i$  and  $E_i$  events were generated in accordance with assigned values of  $\pi_1$ ,  $\pi_2$ ,  $\alpha$ , and the observed  $O_i$  responses.

On all trials, the signal light was lighted for 2 sec. The appropriate  $s_i$  stimulus light immediately followed the cessation of the signal light and remained on for 3 sec. After

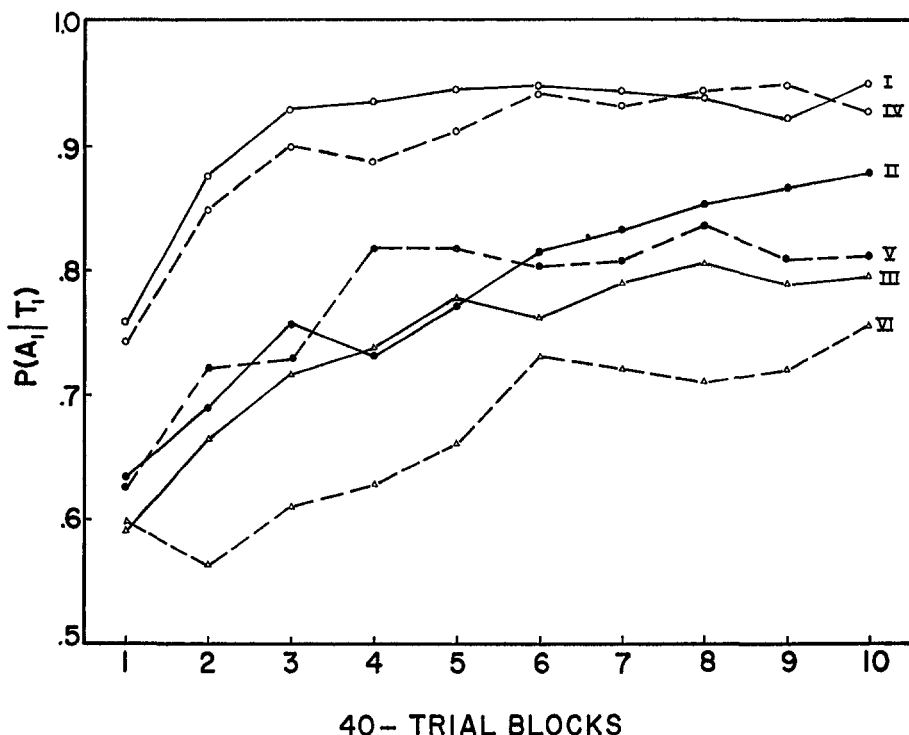


FIG. 1. The average proportion of  $A_1$  responses on  $T_1$ -type trials in successive blocks of 40 trials.

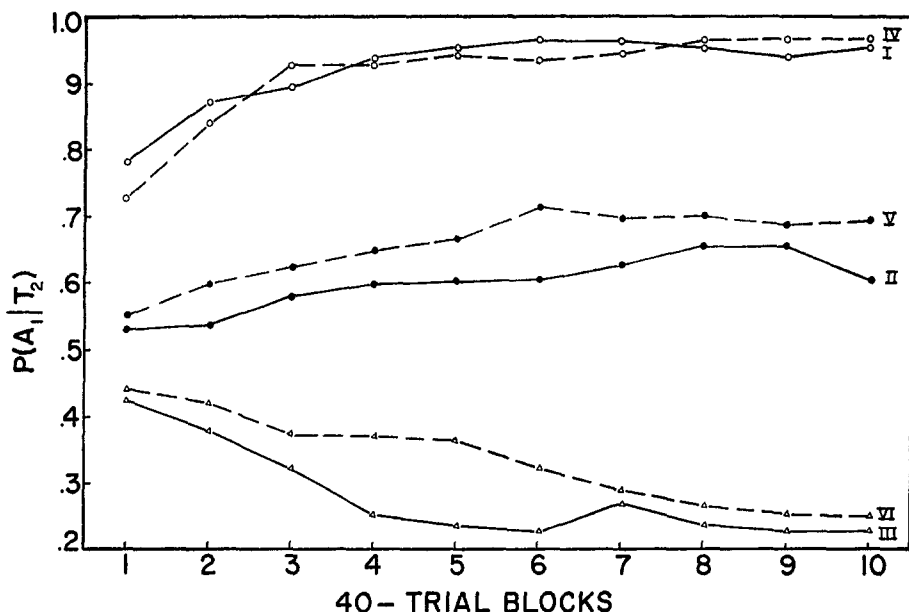


FIG. 2. The average proportion of  $A_1$  responses on  $T_2$ -type trials in successive blocks of 40 trials.

the offset of the  $s_i$  light, one of the reinforcing lights went on for 2 sec. The time between the offset of the reinforcing light and the onset of the signal light for the next trial was 3 sec.

## RESULTS AND DISCUSSION

*Mean learning curves and asymptotic results.*—Figure 1 presents the average proportion of  $A_1$  responses on  $T_1$ -type trials in successive blocks of 40 trials. For each  $S$  the proportion of  $A_1$ 's on  $T_1$  trials was tabulated for a 40-trial block, and these quantities were then averaged over  $S$ s. Similarly, Fig. 2 presents the average proportion of  $A_1$ 's on  $T_2$ -type trials in successive blocks of 40 trials. Finally, Fig. 3 presents the average proportion of  $O_1$  responses. In all three figures the curves appear to be reasonably stable over the last half of the experiment. Consequently,

the proportions computed over the final block of 160 trials were used as estimates of asymptotic performance.

Table 1 presents the observed mean proportions over the last 160-trial block and the related  $SD$ s. The observed values of  $P_\infty(A_1|T_i)$  were computed as indicated in the description of Fig. 1 and 2. The observed values of  $P_\infty(O_i \cap A_1)$  were computed by obtaining, for individual  $S$ s, the proportion of trials on which both the  $A_1$  response and the  $O_i$  response occurred and then averaging over  $S$ s.

The values predicted by the model are also presented in Table 1 for the case where  $\theta' = \theta'' = \theta$ . Expressions for the  $u_k$ 's were derived by standard methods (Feller, 1957), and then combined by Equations 1–5 to predict the response probabilities. The computations were performed at the Western Data Processing Center on

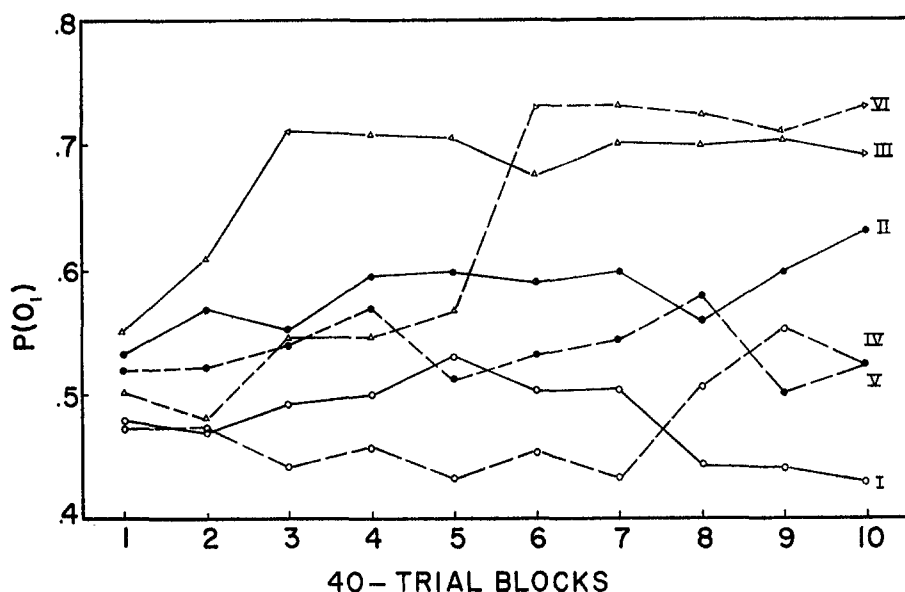
FIG. 3. The average proportion of  $O_1$  responses in successive blocks of 40 trials.

TABLE 1  
PREDICTED AND OBSERVED RESPONSE PROBABILITIES OVER THE LAST BLOCK  
OF 160 TRIALS

	Group I			Group II			Group III		
	Pred.	Obs.	SD	Pred.	Obs.	SD	Pred.	Obs.	SD
$P_{\infty}(A_1 T_1)$	.90	.94	.014	.81	.85	.164	.79	.79	.158
$P_{\infty}(A_1 T_2)$	.90	.94	.014	.59	.61	.134	.21	.23	.182
$P_{\infty}(O_1)$	.50	.45	.279	.55	.59	.279	.73	.70	.285
$P_{\infty}(O_1 \cap A_1)$	.45	.43	.266	.39	.42	.226	.37	.36	.164
$P_{\infty}(O_2 \cap A_1)$	.45	.47	.293	.31	.31	.232	.13	.16	.161
	Group IV			Group V			Group VI		
	Pred.	Obs.	SD	Pred.	Obs.	SD	Pred.	Obs.	SD
$P_{\infty}(A_1 T_1)$	.90	.93	.063	.80	.82	.114	.73	.73	.138
$P_{\infty}(A_1 T_2)$	.90	.95	.014	.60	.68	.114	.27	.25	.138
$P_{\infty}(O_1)$	.49	.50	.257	.52	.53	.305	.63	.72	.263
$P_{\infty}(O_1 \cap A_1)$	.44	.47	.241	.35	.38	.219	.32	.36	.138
$P_{\infty}(O_2 \cap A_1)$	.46	.47	.247	.34	.36	.272	.19	.13	.168

an IBM 709 computer.<sup>2</sup> By presenting a single value for each theoretical

<sup>2</sup> The program or punch program deck is available to anyone interested in generating theoretical results for parameter values not considered in this paper.

quantity in Table 1 we imply that these predicted proportions are independent of  $\theta$ . Actually this is not always the case. However for the event schedules employed in this experiment the dependency of the

theoretical proportions on  $\theta$  is negligible. For  $\theta$  ranging over the interval .00001 to 1.0 the values of the predicted proportions are affected in only the third or fourth decimal place; it is for this reason that we present theoretical values to only two decimal places.<sup>3</sup>

In view of these comments it should be clear that the predictions in Table 1 are based solely on the experimentally assigned values of  $\pi_1$ ,  $\pi_2$ , and  $\alpha$ . Thus, they are entirely a priori and do not make use of any parameters evaluated from the data. Consequently, differences between  $S$ s, which can be represented by inter- $S$  variability in  $\theta$ , do not substantially affect these asymptotic predictions. Of course, this implies that the observed proportions for individual  $S$ s and also proportions averaged over  $S$ s should both approach these predicted values with increasing sample size.

An inspection of Table 1 indicates good agreement between observed and predicted quantities. The observed value of  $P_\infty(A_1|T_1)$  decreases from Groups I to III and also from Groups IV to VI as predicted by the model. Similarly the observed values of  $P_\infty(A_1|T_2)$  decrease from I to III and from IV to VI as expected. Column comparisons are also in the appropriate order, that is, on this measure Group I is less than IV, II is less than V, and III is less than VI. Thus, as we increase the frequency of reinforcing the  $A_2$  response on  $T_2$  trials, we not only observe an increment in  $P_\infty(A_2|T_2)$  but also a decrement in  $P_\infty(A_1|T_1)$ .

For  $P_\infty(O_1)$ , an increase occurs from Groups I to III and from Groups IV to VI in accordance with theoretical results. That is, the propor-

<sup>3</sup> Essentially the same statement holds for  $\theta' \neq \theta''$ . However, in some cases the dependency is slightly larger.

TABLE 2  
ANALYSIS OF VARIANCE OF THE NUMBER OF  $O_1$   
RESPONSES IN THE LAST BLOCK  
OF 160 TRIALS

Source	df	MS	F
$\pi$ (values of $\pi_2$ )	2	27,533.2	13.8*
$\alpha$ values	1	113.4	0.1
$O$ ( $O_1$ left or right)	1	22,253.0	11.1*
$A$ ( $A_1$ left or right)	1	44.2	0.0
Interactions (11)	18		— <sup>a</sup>
Within	216	2,000.7	

<sup>a</sup> None significant at .05 level.

\*  $P < .001$ .

tion of  $O_1$  responses increases as a function of the difference between  $\pi_1$  and  $\pi_2$ ; of course, this result would be expected in view of the fact that differential reinforcement for the observing responses depends on the difference between the reinforcement schedules on  $T_1$  and  $T_2$  trials. However, column comparisons on the  $P_\infty(O_1)$  measure for I–IV and III–VI are in the reverse order; the difference on the  $P_\infty(O_1)$  measure is particularly large for Group VI. This difference between data and theory for Group VI is also reflected in  $P_\infty(O_2 \cap A_1)$ ; in fact, the discrepancies of these two quantities from predicted values are greater than any of the others in Table 1.

An analysis of variance on the number of  $O_1$  responses in the last block of 160 trials is presented in Table 2. The effects of the  $O_1$  and  $A_1$  placements on  $S$ 's panel (i.e., right or left) are included in the analysis. The effect of the  $\pi$ -variable is highly significant as would be expected. However, the  $\alpha$ -variable is not significant. This finding might have been anticipated since the theoretical prediction for the over-all effect of  $\alpha$  is small for the parameter values used in this study. The most unexpected result of the analysis is with regard to the observing response

variable; the placement of the  $O_1$  key turns out to be highly significant while the placement of the  $A_1$  key has no effect. Over all groups and  $S$ s for the last 160 trials, the right hand observing response key was chosen on 55% of the trials while the right hand  $A_j$  key was selected on 50% of the trials. This right position preference on the observing response keys is particularly surprising in view of the fact that no similar preference exists for the  $A_j$  key. Several variables may account for this finding; for example, the observing response keys are in juxtaposition while the  $A_j$  keys are well separated; also, the observing response keys are further from  $S$  than the  $A_j$  keys.

In order to evaluate statistically the adequacy of the present model we have run a test suggested by Pillai and Ramachandran (1954) on the  $P_\infty(O_1)$  measure. The test involves taking the largest absolute difference between an observed mean value and the predicted value in a collection of samples (in this case six). This difference is then divided by an over-all estimate of the standard error of the mean, that is, it is assumed that the observations are randomly selected from populations with homogeneous variance. As noted above, the largest discrepancy on the  $P_\infty(O_1)$  measure occurs for Group VI. The predicted number of  $O_1$ s in the last block of 160 trials was 100.8 and the observed mean value was 115.7. The within-cells term in Table 2 was used to estimate the standard error of the mean, and in terms of Cochran's test there was no reason to reject the assumption of homogeneous variance. The obtained value of the Pillai-Ramachandran statistic was 2.1 and was not significant at the .05 level. Consequently, in terms of this particular statistical criterion there is no evidence to suggest that we reject the present model.

As noted earlier, not only group means but also the responses of individual  $S$ s should approach the theoretical values

presented in Table 1. A check on the correspondence between individual asymptotic behavior and predicted values is equivalent to evaluating the agreement between observed  $SD$ s presented in Table 1 and asymptotic variability predicted by the model. Unfortunately direct computation of the theoretical  $SD$  is extremely cumbersome, and we have not obtained an analytical result. However, research reported by Suppes and Atkinson (1960) dealing with a similar model found that observed  $SD$ s were substantially larger than predicted values. Considering the rather large  $SD$ s reported here, their finding may be applicable to this set of data.

*Transition characteristics.*—A basic assumption in the model requires that if  $S$  is correct on trial  $n$  (i.e.,  $A_1-E_1$  or  $A_2-E_2$  occurs) then on trial  $n + 1$  he will repeat the observing response made on trial  $n$ . However, if  $S$  is incorrect (i.e.,  $A_1-E_2$  or  $A_2-E_1$  occurs) then with probability  $\theta'$  he will shift observing responses from trial  $n$  to  $n + 1$ . This is a strong assumption and yields a highly deterministic set of predictions; for example, repetition of an observing response with probability 1 if  $S$  is correct on the preceding trial. On the other hand, a weaker form of the assumption which requires only a greater probability of observing response alternation following trials on which incorrect as compared to correct discrimination responses occur seems to be a reasonable conjecture for this type of problem. To test this class of assumptions we have computed the proportions of observing response alternations conditionalized on correct and incorrect discrimination responses over the last 160 trials. Let  $N_n(s|c)$  denote the number of  $S$ s who were correct on trial  $n - 1$  and shifted observing responses from trial  $n - 1$  to  $n$ ; also, let  $N_n(c)$  be the number of  $S$ s who were correct



on trial  $n$ . Similarly, define  $N_n(s|\bar{c})$  and  $N_n(\bar{c})$  in terms of incorrect responses. Further, define

$$N(s|c) = \sum_{n=241}^{400} N_n(s|c)$$

and

$$N(c) = \sum_{n=240}^{399} N_n(c)$$

and define  $N(s|\bar{c})$  and  $N(\bar{c})$  similarly. Then estimates of the conditional probabilities of shifting observing responses following correct or incorrect discrimination responses are, respectively,

$$\hat{P}(s|c) = \frac{N(s|c)}{N(c)},$$

$$\hat{P}(s|\bar{c}) = \frac{N(s|\bar{c})}{N(\bar{c})}$$

Table 3 presents the observed data for each of the groups. No statistical test is needed to see that these observed transition probabilities differ significantly from theoretical values. It suffices to note that theoretically  $\hat{P}(s|c)$  should be identically zero for all groups whereas the observed values of this quantity differ markedly from zero. Without regard to the specific assumption considered in this paper, the question can be raised as to whether or not shifting of an observing response is more likely following incorrect or correct trials, that is whether  $\hat{P}(s|\bar{c})$  is greater than  $\hat{P}(s|c)$ . A formal test of this hypothesis is a complex matter and we do not attempt it here. However note that for five of the six groups  $\hat{P}(s|\bar{c})$  is greater than  $\hat{P}(s|c)$ . Further, the difference between these quantities increases as  $\pi_2$  decreases; that is, the difference increases from Groups I to III and from Groups IV to VI.

TABLE 3  
TRANSITION FREQUENCIES AND ESTIMATED  
PROPORTIONS OVER THE LAST BLOCK  
OF 160 TRIALS

	Groups					
	I	II	III	IV	V	VI
$N(s c)$	1610	880	933	1617	883	735
$N(c)$	5388	4080	4664	5463	4032	4350
$N(s \bar{c})$	275	520	446	266	517	524
$N(\bar{c})$	973	2280	1696	897	2328	2010
$\hat{P}(s c)$	.299	.216	.200	.296	.219	.169
$\hat{P}(s \bar{c})$	.283	.228	.263	.297	.222	.261

In conclusion, the rather striking correspondence between theoretical and observed values in Table 1 lends considerable support to the main features of the model. For the type of discrimination problem considered in this paper, it seems clear that asymptotic behavior can be predicted with accuracy in terms of the particular relations we have postulated among reinforcement schedules, observing responses, and discrimination responses. However, the sequential data reported in Table 3 indicate that some of the detailed features of the stimulus sampling process assumed in the model are certainly incorrect; this finding is not too surprising in view of related research on similar Markov chain models. Fortunately, within the framework of stimulus sampling theory, one can restate our axioms in only slightly modified form and thereby avoid the completely deterministic predictions made by the present model for sequential data. The disadvantage of such a reformulation is that the mathematical complexity of the model is greatly increased. The reader interested in details of such modifications is referred to Suppes and Atkinson (1960).

### SUMMARY

An analysis of observing responses in discrimination learning was made. The typical discrimination task was modified so that two mutually exclusive and exhaustive observing responses could be identified and directly recorded. The experimental situa-

tion involved a series of 400 trials, each trial belonging to one of two types ( $T_1$  or  $T_2$ ). The sequence of events on a trial was as follows: (a) ready signal to which  $S$  made an observing response; (b) the presentation of one of three stimuli; (c) occurrence of one of two discrimination responses to the stimulus presentation; (d) termination of the trial with the reinforcement of a discrimination response. The particular stimulus presented on a trial depended on the observing response and the trial type. Following one of the observing responses, different stimuli were presented on  $T_1$  and  $T_2$  trials so that it was possible for  $S$  to identify the trial type; following the other observing response, the same stimulus was presented on both types of trials and hence  $S$  could not identify the trial type.

Six groups of college students were tested. The major independent variable specified different pairs of reinforcement schedules for the two trial types. The results indicated a highly predictable relation between the selection of observing responses and reinforcement schedules. In general, the greater the difference between the reinforcement schedules on  $T_1$  and  $T_2$  trials, the greater the preference for one observing response over the other. The analysis of the data was in terms of a Markov chain model which is closely related to stimulus sampling theory. There was excellent agreement between theoretical and observed values on asymptotic measures of observing and discrimination responses. However, an analysis of the sequential data indicated certain difficulties with the model.

#### REFERENCES

- ATKINSON, R. C. A Markov model for discrimination learning. *Psychometrika*, 1958, **23**, 309-322.
- ATKINSON, R. C. A theory of stimulus discrimination learning. In K. J. Arrow, S. Karlin, & P. Suppes (Eds.), *First Stanford symposium on mathematical methods in the social sciences*. Stanford: Stanford Univer. Press, 1960. Ch. 15.
- ATKINSON, R. C., BOGARTZ, W. H., & TURNER, R. N. Supplementary Report: Discrimination learning with probabilistic reinforcement schedules. *J. exp. Psychol.*, 1959, **57**, 349-350.
- BURKE, C. J., & ESTES, W. K. A component model for stimulus variables in discrimination learning. *Psychometrika*, 1957, **22**, 133-145.
- ESTES, W. K. Toward a statistical theory of learning. *Psychol. Rev.*, 1950, **57**, 94-107.
- ESTES, W. K., & BURKE, C. J. A theory of stimulus variability in learning. *Psychol. Rev.*, 1953, **60**, 276-286.
- FELLER, W. *An introduction to probability theory and its applications*. (2nd ed.) New York: Wiley, 1957.
- PILLAI, K. C. S., & RAMACHANDRAN, K. V. On the distribution of the ratio of the  $i^{\text{th}}$  observation in an ordered sample from a normal population to an independent estimate of the standard deviation. *Ann. math. Statist.*, 1954, **25**, 565-572.
- RESTLE, F. A survey and classification of learning models. In R. R. Bush & W. K. Estes (Eds.), *Studies in mathematical learning theory*. Stanford: Stanford Univer. Press, 1959. Ch. 20.
- SUPPES, P., & ATKINSON, R. C. *Markov learning models for multiperson interactions*. Stanford: Stanford Univer. Press, 1960.
- WYCKOFF, L. B., JR. The role of observing responses in discrimination behavior. *Psychol. Rev.*, 1952, **59**, 431-442.

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